

Competition between standard and exotic double beta decays

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Abstract

We discuss the contributions of higher order terms in weak Hamiltonian to the standard two-neutrino double beta decay. The formalism for the unique first forbidden transitions has been developed, and it is shown that they can alter the two-electron energy spectrum. Yet, their effect is too small to screen the detection of exotic neutrinoless double beta decays, which are candidates for testing the physics beyond the standard model.

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The double beta ($\beta\beta$) experiments furnish a unique window onto whatever new physics may replace the Standard Electroweak Model (SM). To be observable the new physics should: i) violate the electron-lepton-number (L_e) conservation, that is fulfilled in the SM, and/or ii) fit scalar particles (currently called Majorons), light enough to be produced in the $\beta\beta$ decay.

The quantity that is used to discern experimentally between the ordinary SM two-neutrino decays ($\beta\beta_{2\nu}$) and the exotic neutrinoless $\beta\beta$ events, both without ($\beta\beta_{0\nu}$) and with Majoron emissions ($\beta\beta_M$), is the electron energy spectrum $d\Gamma/d\epsilon$ of the decay rate Γ , as a function of the sum $\epsilon = \epsilon_1 + \epsilon_2$ of the energies of the two emitted electrons. The $\beta\beta_{2\nu}$ decay exhibit a continuous spectra in the interval $2 \leq \epsilon \leq Q$, where $Q = E_i - E_f$ is the released energy. On the other hand, when no new light particles are created, the L_e -violating terms in the weak Lagrangian, that generate a Majorana mass for the neutrino, can be identified if they produce the $\beta\beta_{0\nu}$ decay, with the energy spectrum that is just a spark at the energy Q . The spectra for the $\beta\beta_M$ decays are also continuous, but their shapes are clearly differentiated from that for the $\beta\beta_{2\nu}$ decay, and depend on whether one or two Majorons are emitted and on the leptonic charge ($L_e = 0, -1, -2$) they carry [1, 2, 3]. Thus, both $\beta\beta_{0\nu}$ and $\beta\beta_M$ processes, that are potentially capable to reveal the new physics, are clearly distinguishable from the SM $\beta\beta_{2\nu}$ decay.

The sensitivity of the $\beta\beta$ decay experiments is steadily and constantly increasing. For instance, while the pioneer laboratory measurement of the $\beta\beta_{2\nu}$ decay in ^{82}Se has relied on only 40 events [4], the most recent experiment on ^{76}Ge [5] was done with high statistics (~ 20000 counts). Another example is the evolution of the half life limit for the $\beta\beta_{0\nu}$ decay in ^{76}Ge . From the first measurement in 1952, $T_{0\nu} > 2 \cdot 10^{16}$ y [6], it has varied to $T_{0\nu} > 2.2 \cdot 10^{22}$ y in 1983 [7], while the most recent value is $T_{0\nu} > 1.2 \cdot 10^{25}$ y [5]. By comparing the last one, as well as the measured half life limit for $\beta\beta_M$ decay: $T_M > 1.67 \cdot 10^{22}$ y, with the corresponding half life for $\beta\beta_{2\nu}$ decay: $T_{2\nu} \cong 1.77 \cdot 10^{21}$ y, it can be said that presently are being observed effects of the order of 10^{-4} at $\epsilon \sim Q$ and of the order of 10^{-1} at $\epsilon \sim Q/2$. There are also several ongoing and planned experiments that are supposed to allow for measurements of still smaller effects. The most promising one seems the GENIUS project [5], which is supposed to test the $\beta\beta_{0\nu}$ half-life of ^{76}Ge up to a limit of $T_{0\nu} > 5.7 \cdot 10^{28}$ y. One might expect that the sensitivity for probing the $\beta\beta_M$ decay will be improved accordingly as well.

The measured limits on the exotic $\beta\beta$ transition probabilities are being rapidly translated into more stringent constraints on the parameters of new theoretical developments in particle physics, such as: Majorana mass of the light neutrinos, right-handed weak couplings, right-handed weak coupling involving heavy Majorana neutrinos, massless Majoron emission, R-parity breaking in the supersymmetric model, *etc.* They have also broad consequences on the history of the primordial universe, evolution of stellar object and astrophysics of supernovas.

In confronting the experimental data with theory, the validity of allowed approximation (A) is usually assumed for the standard $\beta\beta_{2\nu}$ decay. This implies to consider only virtual states with spin and parity $J^\pi = 0^+$ and 1^+ , which contribute via the nuclear operators $g_V \tau^+$ and $g_A \tau^+ \sigma$, respectively. The higher order effects, coming from the parity-forbidden (PF) virtual states with $J^\pi = 0^-, 1^-, 2^-$, have been ignoring almost entirely all along by workers in the field, simply because

they expected them to be small. But, in planing future experiments and in searching for exotic decays, it might be important to know how small these effects are, and whether they could eventually lead to experimental consequences similar to that produced by the former.

Besides, in the charged Majoron (CM) model designed by Burgess and Cline [1] and by Carone [2], which is the most hopeful one to be observed experimentally among the new Majoron models [8], the $\beta\beta$ decay proceeds via relativistic corrections in the hadronic current. More, the nuclear matrix element is of the form $\mathcal{M}_{CM} = \mathcal{M}_{CM}^+ - \mathcal{M}_{CM}^-$, with \mathcal{M}_{CM}^\pm being the contributions of two heavy Dirac neutrinos with masses M_\pm , and there is a strong destructive interference between \mathcal{M}_{CM}^+ and \mathcal{M}_{CM}^- when $M_+ \cong M_-$ [9]. Therefore it might be also interesting to compare the outcome of the CM model with that arising from the PF transitions in the standard $\beta\beta$ decay.¹

Naively thinking it could be inferred that the PF contributions to the $\beta\beta_{2\nu}$ amplitude are of the order of $(RQ/4)^2$, where R is the nuclear radius and the factor 4 comes from the fact that the Q -value is shared by 4 leptons. Therefore, as for medium heavy nuclei $(RQ/4)^2 \sim 10^{-3}$, they would alter the half lives and spectrum shapes at the level of 10^{-3} or 10^{-6} , depending on whether there are interference or not between the A and PF contributions, *i.e.* whether the PF matrix elements enter linearly or only quadratically in the expression for the decay rate.

The above estimate, however, it is not valid for the non-unique (NU) transitions with $\Delta J^\pi = 0^-, 1^-$, due to: i) the electron s -waves contributions via the velocity dependent terms in the hadronic current, and ii) the Coulomb enhancement of the matrix elements coming from the $p_{1/2}$ waves. When these effects are considered, and because the interference between the NU and A matrix elements, the theoretical half lives are increased in $\sim 30\%$ [11, 12]. Yet, due to the nuclear structure uncertainties, we cannot disentangle from the measured half lives alone how large is the effect of the NU matrix elements, even when the experimental errors are relatively small.

We have also pointed out [12] that the NU transitions do not modify the allowed shape of the two-electron spectrum, and that the unique (U) transitions states should be examined. It should be remembered that the spectrum shape of a single β transition of this type, provides for the emission of more high- and low-energy electrons, than are found in spectra that have the allowed shape [13, 14]. Thus, it sounds appropriate to speculate about a similar effect in the $\beta\beta$ decay. Simultaneously, it is essential to find out whether the U matrix elements contribute linearly or quadratically to the $\beta\beta_{2\nu}$ decay rate.

Meanwhile, Civitarese and Suhonen [15], have pointed out that the effect of the U transitions on the $T_{2\nu}$ in ^{76}Ge can be disregarded, but they have not discussed at all the corresponding spectrum shape. Without performing any theoretical development, it is simply assumed that the U matrix element come through quadratically, and that the ratio between the phase-space factors, for A and U $\beta\beta_{2\nu}$ transitions, is of the order of 10^6 . Besides, in the same work it is suggested that the NU transitions should be retarded as well by the factor $(RQ/4)^4 \sim 10^{-6}$.

Before presenting the numerical results for the $\beta\beta_{2\nu}$ decay through the virtual states $J^\pi = 2^-$,

¹In fact, except for the neutrino mass term, the $\beta\beta_{0\nu}$ decay also arises from the higher order effects [10].

than its total energy, which leads to the ξ -approximation [12, 14]. Thus, for the purposes of the present study, and after a lengthy algebra, we cast the weak Hamiltonian in a rather novel form

$$H_w(e\nu) = -\frac{G}{2} \sum_{\pi J} \mathbf{W}_J^\pi \cdot \mathbf{L}_J(e\nu), \quad (6)$$

which can be used for multiple purposes. Here \mathbf{W}_J^+ and \mathbf{W}_J^- are, respectively, the allowed and forbidden nuclear operators, and

$$\mathbf{L}_J(e\nu) = sg(s_\nu) \sqrt{\frac{\epsilon+1}{2\epsilon}} F_0(\epsilon) \chi^\dagger(s_e) \left(1 - \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{\epsilon+1}\right) \ell_J (1 - \boldsymbol{\sigma} \cdot \hat{\mathbf{q}}) \chi(-s_\nu), \quad (7)$$

are the leptonic matrix elements, with $\chi(s)$ being the usual Pauli spinor. The leptonic operators ℓ_J are listed in Table 1, together with \mathbf{W}_J^π .

Table 1: Operators ℓ_J and \mathbf{W}_J^π for different multipoles J ; $\bar{p} = p[F_1(\epsilon)/F_0(\epsilon)]^{1/2}$, $\mathbf{v} = \mathbf{p}/M_N$ and $\xi = \alpha Z/2R$.

J	ℓ_J	\mathbf{W}_J^+	\mathbf{W}_J^-
0	1	g_V	$-g_A(\boldsymbol{\sigma} \cdot \mathbf{v} + \xi i \boldsymbol{\sigma} \cdot \mathbf{r})$
1	$\boldsymbol{\sigma}$	$g_A \boldsymbol{\sigma}$	$-g_V \mathbf{v} - \xi [g_V i \mathbf{r} - g_A(\boldsymbol{\sigma} \times \mathbf{r})]$
2	$[\boldsymbol{\sigma} \otimes (\mathbf{q} + \bar{\mathbf{p}})]_2$	-	$ig_A(\boldsymbol{\sigma} \otimes \mathbf{r})_2/\sqrt{5}$

In the next step we evaluate the transition amplitude and get

$$R_{2\nu} = \frac{G^2}{4(2\pi)^6} [1 - P(\nu_1\nu_2)] \left[\mathbf{L}_0(e_1\nu_1) \cdot \mathbf{L}_0(e_2\nu_2) (\mathcal{M}_{2\nu}^A + \mathcal{M}_{2\nu}^{NU}) - \mathbf{L}_2(e_1\nu_1) \cdot \mathbf{L}_2(e_2\nu_2) \mathcal{M}_{2\nu}^U \right], \quad (8)$$

where $\mathcal{M}_{2\nu}^A = \mathcal{M}_{2\nu}(0^+) + \mathcal{M}_{2\nu}(1^+)$, $\mathcal{M}_{2\nu}^{NU} = \mathcal{M}_{2\nu}(0^-) + \mathcal{M}_{2\nu}(1^-)$, and $\mathcal{M}_{2\nu}^U \equiv \mathcal{M}_{2\nu}(2^-)$ are, respectively, the $\beta\beta_{2\nu}$ matrix elements for the A, NU and U transitions, and

$$\mathcal{M}_{2\nu}(J^\pi) = \sum_{\alpha} (-1)^J \frac{\langle 0_F^+ || \mathbf{W}_J^\pi || J_\alpha^\pi \rangle \langle J_\alpha^\pi || \mathbf{W}_J^\pi || 0_I^+ \rangle}{E_{J_\alpha^\pi} - E_{0_I^+} + Q/2}. \quad (9)$$

After introducing (8) into (1) and performing the spin summations and angular integration, the contribution of the lepton matrix elements $\mathbf{L}_2(e_1\nu_1) \cdot \mathbf{L}_2(e_2\nu_2) \mathbf{L}_0^*(e_1\nu_i) \cdot \mathbf{L}_0^*(e_2\nu_j)$ turns out to be identically null for $i, j = 1, 2$ or $2, 1$. Therefore, *there is no interference term between the A and U matrix elements.*, as happens with $\mathcal{M}_{2\nu}^A$ and $\mathcal{M}_{2\nu}^F$. We get

$$d\Gamma_{2\nu} \equiv d\Gamma_{2\nu}^{A+NU} + d\Gamma_{2\nu}^U = \frac{4G^4}{15\pi^5} \left[|\mathcal{M}_{2\nu}^A + \mathcal{M}_{2\nu}^{NU}|^2 d\Omega_{2\nu}^A + |\mathcal{M}_{2\nu}^U|^2 d\Omega_{2\nu}^U \right], \quad (10)$$

where

$$d\Omega_{2\nu}^A = \frac{1}{2^6 \pi^2} (Q - \epsilon_1 - \epsilon_2)^5 \prod_{k=1}^2 p_k \epsilon_k F_0(\epsilon_k) d\epsilon_k, \quad (11)$$

is the usual phase space for the $\beta\beta_{2\nu}$ in the A approximation, and

$$d\Omega_{2\nu}^U = \frac{5^2}{2^{11} 3^2 \pi^2} \sum_{i=0}^2 a_i (Q - \epsilon_1 - \epsilon_2)^{5+2i} \prod_{k=1}^2 p_k \epsilon_k F_0(\epsilon_k) d\epsilon_k, \quad (12)$$

with $a_0 = \bar{p}_1^2 \bar{p}_2^2$, $a_1 = 16\bar{p}_1^2/35$ and $a_2 = 1/21$.

The corresponding half life is

$$T_{2\nu}(0_I^+ \rightarrow 0_{NU}^+) = \left(\mathcal{G}_{2\nu}^A |\mathcal{M}_{2\nu}^A + \mathcal{M}_{2\nu}^{NU}|^2 + \mathcal{G}_{2\nu}^U |\mathcal{M}_{2\nu}^U|^2 \right)^{-1}, \quad (13)$$

where

$$\mathcal{G}_{2\nu}^{A,U} = \frac{4G^4}{15\pi^5 \ln 2} \int d\Omega_{2\nu}^{A,U}, \quad (14)$$

are the kinematical factors.

The spectrum shapes $d\Gamma_{2\nu}^{A+NU}/d\epsilon$ and $d\Gamma_{2\nu}^U/d\epsilon$ for ^{76}Ge are confronted in Fig. 1. At variance with the single β emission, *the spectrum for the U double beta process deviates from the allowed shape in the low-energy region, but not for $\epsilon \cong Q$* . Thus, independently of magnitude of $\Gamma_{2\nu}^U$, the virtual states $J^\pi = 2^-$ will never interfere with the detection of the $\beta\beta_{0\nu}$ events. Their spectra, still, can overlap with those engendered by the $\beta\beta_M$ decays. This is illustrated in same figure for the case of the CM model.

Table 2: Kinematical factors $\mathcal{G}_{2\nu}$, and the nuclear matrix elements $\mathcal{M}_{2\nu}$ evaluated within the QRPA formalism.

Nucleus	$\mathcal{G}_{2\nu}^A [y^{-1}]$	$\mathcal{G}_{2\nu}^U [y^{-1}]$	$\mathcal{M}_{2\nu}^A$	$\mathcal{M}_{2\nu}^{NU}$	$\mathcal{M}_{2\nu}^U$
^{76}Ge	$5.39 \cdot 10^{-20}$	$2.10 \cdot 10^{-19}$	0.050	-0.008	$1.0 \cdot 10^{-5}$
^{82}Se	$1.80 \cdot 10^{-18}$	$2.54 \cdot 10^{-17}$	0.060	-0.009	$9.8 \cdot 10^{-6}$
^{100}Mo	$3.91 \cdot 10^{-18}$	$5.50 \cdot 10^{-17}$	0.051	-0.014	$1.1 \cdot 10^{-5}$

Numerical results for the kinematical factors and the nuclear matrix elements, for several experimentally interested nuclei, are displayed in Table 2. The moments $\mathcal{M}_{2\nu}$ were evaluated within the pn-QRPA model, following the procedure adopted in our previous works [12, 17]. It can be easily seen that:

$$|\mathcal{M}_{2\nu}^U| \cong R^2 |\mathcal{M}_{2\nu}^A + \mathcal{M}_{2\nu}^{NU}|; \quad \mathcal{G}_{2\nu}^U \cong (Q/4)^4 \mathcal{G}_{2\nu}^A.$$

Therefore, from the theoretical developments and numerical calculations done here, it can be stated that the simple estimate

$$T_{2\nu}^U/T_{2\nu}^{A+NU} \cong (RQ/4)^{-4} \sim 10^6,$$

is appropriate for the unique transitions.

Table 3: Calculated half-lives (in units of y) for the A+NU and U transitions and for the charge Majoron emission.

Nucleus	$T_{2\nu}^{A+F}$	$T_{2\nu}^U$	$T_{CM}(M_+ \rightarrow \infty)$	$T_{CM}(M_+ \cong M_-)$
^{76}Ge	$1.1 \cdot 10^{22}$	$4.7 \cdot 10^{28}$	$1.6 \cdot 10^{25}$	$1.3 \cdot 10^{29}$
^{82}Se	$2.1 \cdot 10^{20}$	$4.1 \cdot 10^{26}$	$9.2 \cdot 10^{23}$	$9.5 \cdot 10^{27}$
^{100}Mo	$1.9 \cdot 10^{20}$	$1.4 \cdot 10^{26}$	$3.8 \cdot 10^{23}$	$4.1 \cdot 10^{27}$

Finally, in Table 3 are compared the half-lives for the $\beta\beta_{2\nu}$ and $\beta\beta_{CM}$ decays. In the CM model the effective coupling constant was taken to be $g_{CM} = \theta^2/2$ with $\theta = 0.1$, and two different values for the heavy Dirac neutrinos masses were considered, namely $M_+ \rightarrow \infty$ and $M = 100$ MeV, and $M_{\pm} = M\sqrt{1 \pm \theta}$, with $M = 100$ MeV [1, 9]. It turns out that:

$$T_{CM}(M_+ \rightarrow \infty)/T_{2\nu}^{A+NU} \sim 10^3; \quad T_{CM}(M_+ \cong M_-)/T_{2\nu}^{A+NU} \sim 10^7.$$

Thus, the emission rate for the recently discovered Majoron models is very strongly conditioned by the model parameters, and it could be so small as that arising from the unique forbidden transitions.

In summary, the higher order effects in the standard physics modify the $\beta\beta_{2\nu}$ spectrum shape but only at the level of 10^{-6} and mainly at low two-electron energy, where most backgrounds tend to dominate. Therefore they would hardly mask the observation of the potential exotic $\beta\beta$ decays.

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Figure Captions

Fig. 1. Electron energy spectrum for the nucleus ^{76}Ge , as a function of the sum of energies of the two emitted electrons, for: the standard 2ν allowed ($\beta\beta_{2\nu}^A$) and unique forbidden transitions ($\beta\beta_{2\nu}^U$), and the exotic neutrinoless decays, with Majoron charged emission ($\beta\beta_{CM}$) and without ($\beta\beta_{0\nu}$). All four curves have been arbitrarily assigned the same maximal values for purposes of comparison.